Global Imbalances: Exchange Rate Test

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Abstract

We study implications of the global imbalances model that relies on differences in the amount of insurable idiosyncratic risk. Plausibly parameterized model successfully accounts for the size of global imbalances and a 1/3 of decline of the world real interest rate.

We extend this model to a two-good environment and evaluate its predictions for real exchange rates. According to the model the observed imbalance between the U.S. and China implies a 8.6% percent appreciation of the Chinese real exchange rate.

1 Facts

We analyze the period from 1998 until 2007. This is the first ten years after China liberalized capital flows. These are also the years of phenomenal growth in China. While per capita income grew almost tenfold this growth was distributed very unevenly. At the same time Chinese savings, as a percentage of GDP, increased from 20 to 30 percent. This increase in savings outpaced an increase investment and contributed to a massive current account surplus. This surplus was mostly lent to the U.S. via purchases of the U.S. government debt. During the period the U.S. accumulated a net foreign asset position against China equal to -4.64% of the U.S. GDP (see figure 1).

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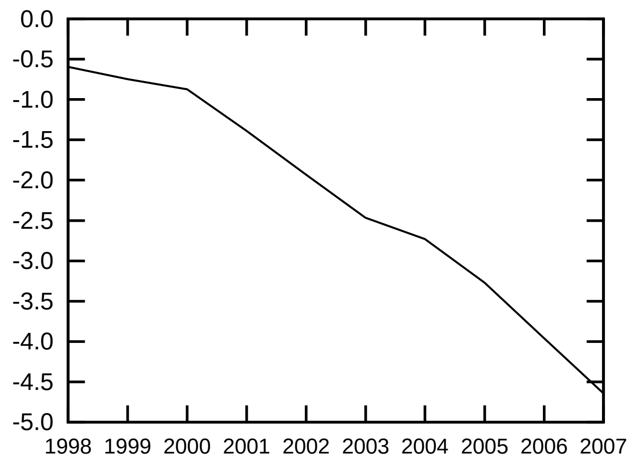

Figure 1: Net foreign asset position between the U.S. and China

The usual suspect is the credit market which still remains very undeveloped in China. Yet, even though there was no marked improvement in the credit market during the decade its condition certainly did not deteriorate. If it was the sole driving force then savings at least must not have increased. Yet we observe the reverse. While we believe that credit market imperfections are a part of the problem we aim to explain the evolution of global imbalances. Here we fix financial market development which is consistent with the evidence. A common measure of financial development, domestic credit to GDP ratio, was essentially unchanged during the period at a level of approximately 110% (see figure 2).

We believe that the driving force is an increase in idiosyncratic uncertainty in China and other fast growing emerging market economies. [REPORT EVIDENCE or relate to Mendoza-Quadrini-RiosRull.]

An increase in idiosyncratic uncertainty increases precautionary demand for savings in China. To keep the world financial market in equilibrium the interest rate declines to contain the savings demand in China. The lower interest rate, however, leads the U.S. into a position of a permanent debtor.

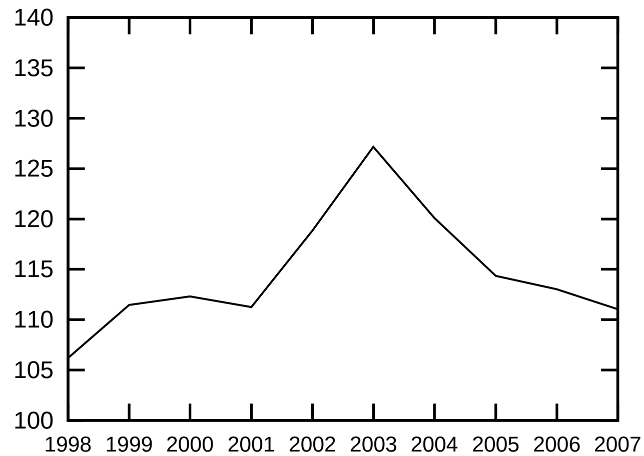

Figure 2: Domestic credit to GDP ratio, %

Figure 3 plots the real interest rate in the US.[1] In 1998 the real interest rate was 3.35% and in 2007 it declined to 1.99%. Decline of the short term real interest rate is consistent with the model.

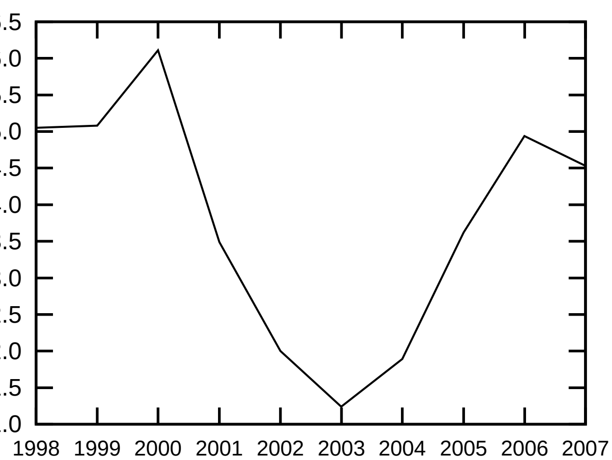

Figure 3: 1-year Treasury bill real rate (constant maturity)

[1] It is computed as the difference between return (constant maturity) on a 1-year T-Bill rate and year-on-year change in the consumer price index (all items).

2 Model

The model described in this paper is a two-country version of the model in Aiyagari (1994). The world consists of two economies each populated by a large number (a continuum) of heterogeneous agents.[2] Economies are indexed by $j \in 1, 2$. Population in country j denoted by $L^j > 0$ and it is fixed exogenously.

Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$.

Uncertainty and endowments. There is no aggregate uncertainty. The only source of uncertainty is household's income. Income shocks are household and country specific. Income of household h in country j is an AR(1) process:

$$ln(y_t^{hj}) = \mu_j(1 - \rho_j) + \rho_j ln(y_{t-1}^{hj}) + \sigma^j e_t^{hj}, \quad e_t^{ij} \sim i.i.d.\ \mathrm{N}(0, 1). \qquad (1)$$

μ_j is the expected individual log-income, which is also country specific.

Preferences of household h in country j are represented by a standard time-separable welfare function:

$$W(c^{hj}) = E\Big[\sum_{t=0}^{\infty} \beta^t u(c_t^{hj})\Big|\mathcal{I}_0\Big], \quad \beta \in (0, 1), \qquad (2)$$

where $\mathcal{I}_t$ denotes the date-t information set. Expectation is over sequences of income realizations.

Financial markets are incomplete. Households in each country can borrow and lend at risk-free gross interest rate R_{t+1}. Flow of capital between countries is unrestricted; so, the interest rate is equalized across borders. Individuals in country j can borrow at most $B^j > 0$.

The initial distribution of financial assets across households in economy j is denoted by $\Omega_0^j(a)$.

Budget constraint of household h in country j is:

$$c_t^{hj} + a_{t+1}^{hj}/R_{t+1}^j = y_t^{hj} + a_t^{hj}, \quad \forall t.$$

[2] The model can be easily extended to the case with a finite number of economies.

2.1 Recursive formulation

All households in each country are identical ex ante and ex post differ only by the paths of realized income. Let $V^j(a, y|R, \sigma)$ be the life-time utility of a household in country j that behaves optimally, has a bank account balance a, current income y. Arguments R, σ reflect the fact that these values are parameters of the individual optimization problem. The value function must solve the following Bellman equation:

$$V^j(a, y|R) = \max_{c \geqslant 0, a' \geqslant -B^j} \left[u(c) + \beta \int V(a', y'|R) dF(y'|y) \right] \tag{3a}$$

subject to

$$c + a'/R = a + y. \tag{3b}$$

The first-order necessary and sufficient condition for the above optimization problem is:[3]

$$u'(c) = \beta R \int u'(c') dF(y'|y) + \mu R, \tag{4}$$

where μ is the Lagrange multiplier on the borrowing constraint. The Lagrange multiplier must satisfy the following complementarity condition:

$$\mu(a' - B^j) = 0.$$

For a given interest rate R, we denote the solution to the country j household's optimization problem by $\rho^j(a, y|R) \in (\mathcal{A} \times \mathcal{Y})^{\mathcal{A} \times R_+}$:[4]

$$a' = \rho^{aj}(a, y|R), \tag{5a}$$

$$c = \rho^{cj}(a, y|R). \tag{5b}$$

2.2 Equilibrium

Let $F_t^j(a, y)$ be a distribution over financial wealth-income pairs in country j:

$$F_t^j(a, y) = prob(a_t^{hj} < a \ and \ y_t^{hj} < y). \tag{6}$$

[3] It will be later shown that the objective is strictly concave. Hence, the above conditions are also sufficient. The budget constraint and the complementary slackness conditions were left out for clarity.

[4] We denote the set of all functions mapping X into Y by X^Y.

Let A_t^j denote the aggregate savings in the economy j in period t when the interest rate equals R. A_t^j is the result of household decisions:

$$A_t^j = \int \rho_{at}^j(a, y) F^j(da, dy). \tag{7}$$

We can now define a concept of competitive equilibrium. We provide a general definition allowing for transitory dynamics.

Definition. An ***autarkic competitive equilibrium*** *in the economy j with the initial distribution of financial wealth $\Omega_0^j(a)$ is a sequence of interest rates $\{R_{t+1}^j\}_{t=0}^{\infty}$ and a sequence of policy functions $\{\rho_{at}^j, \rho_{ct}^j\}_{t=0}^{\infty}$ such that:*

a) Given the sequence of interest rates $\{\rho_{at}^j(a, y), \rho_{ct}^j(a, y)\}$ are the optimal period-t saving and consumption policies of a household living in country j with assets a and current income level y;

b) Financial markets clear:

$$A_t^j = 0, \quad \forall t. \tag{8}$$

Definition. A ***world competitive equilibrium*** *with the initial distribution of financial wealth $\{\Omega_0^1(a), \Omega_0^2(a)\}$ is a sequence of interest rates $\{R_{t+1}\}_{t=0}^{\infty}$ and a sequence of policy functions $\{\rho_{at}^1, \rho_{ct}^1, \rho_{at}^2, \rho_{ct}^2\}_{t=0}^{\infty}$ such that:*

a) Given the sequence of interest rates $\{\rho_{at}^j(a, y), \rho_{ct}^j(a, y)\}$ are the optimal period-t saving and consumption policies of a household living in country j with assets a and current income level y;

b) Financial markets clear:

$$A_t^1 + A_t^2 = 0, \quad \forall t. \tag{9}$$

Definition. A ***stationary autarkic/world competitive equilibrium*** *is an autarkic/world competitive equilibrium in which $R_{t+1}^j = R, \forall t$.*

2.3 Properties of aggregate savings

The aggregate saving function $A(R, \sigma)$ is decreasing in R and σ for all $R \in [0, 1/\beta)$ and $\sigma > 0$.

Proposition 1. *Let R^j denote the autarkic interest rate in country j. If*
a) $\sigma^1 > \sigma^2$ and $B^1 = B^2$ or
b) $\sigma^1 = \sigma^2$ and $B^1 < B^2$
then $R^1 < R < R^2$ and $A^1 > 0 > A^2$.

Proof. See appendix. □

3 Benchmark Simulation

We think of country 1 as representing the U.S. The stochastic process for individual income in country 1 is that estimated by Heaton and Lucas (1995) for a representative U.S. household. Households in country 1 can borrow up to the average annual income. The same borrowing limit was assumed by Huggett (1993). The risk aversion parameter γ was set to a standard value of 2. We choose the discount factor β such that the interest rate in country 1 were 3.35% in the absence of financial linkages with country 2. That is our parameters are set to mimic the situation when China had strict financial account restrictions set in place and any imbalances between the U.S. and China were absent.

Preference parameters in country 2 are the same as in country 1. Country 2 also faces the same *relative* borrowing limit that equals the average period income. Income process in country 2 is also the same with the exception that it is *relatively more volatile.* If individual income had the same coefficient of variation as in country 2 then there would be no trade. In the latter case one country would be just a scaled replica of the other. We set the coefficient of variation for country 2 to 0.4459 (compare to 0.2960 for country 1) so that the interest rate in the stationary equilibrium is 1.99% as observed in 2007.

The relative size of the two economies were chosen to equal the relative size of China versus the U.S. in 1998. We used the data in Penn World Tables version 6.4. In 1998 the U.S. per capita income was 9 times that in China but

Table 1: Benchmark parameters

Parameter	Value	Source
β	0.940	Implies $r = 3.35\%$ in a closed economy
γ	2.000	–
$E(y^1)$	1.000	Normalization
$E(y^2)$	0.110	PWT 6.3
L_1	1.000	Normalization
L_2	4.530	PWT 6.3
ρ_1	0.530	Heaton and Lucas (1995)
σ_1	0.296	Heaton and Lucas (1995)
ρ_2	0.530	Assumed $= \rho_1$
σ_3	0.049	Assumed to satisfy $cv(y_1) = cv(y_2)$
B_1	$1 \times E(y^1)$	Huggett 1993
B_2	$1 \times E(y^2)$	Assumed to satisfy $B_1/E(y_1) = B_2/E(y_2)$

China 4.53 times more populous at the time. Note that without differences in (relative) volatility of individual income country size differences play no role in generating financial imbalances.

Parameter values are summarized in table 1.

Figure 4 demonstrates the solution to the model. It plots aggregate savings of each country as a function of the interest rate R. The equilibrium interest rate is such that world savings is zero. In such an equilibrium country with less risk (lower coefficient of variation of income)[5] will be a net debtor and the other economy a net creditor. Under the parametrization summarized in table 1 country 1 borrows an (astonishing) 32.5% of its GDP from country 2 and the equilibrium net interest rate equals the target 1.99%.

Since we do not have direct evidence on volatility of income in China, we consider next how the size of global imbalance changes with σ^2. Figure 5 panel (a) and (b) plot respectively the country 2's savings and the equilibrium interest rate as a function of σ^2. Benchmark calibration is marked with

[5] It could also be a country with a more developed financial system as reflected in a more generous borrowing limit.

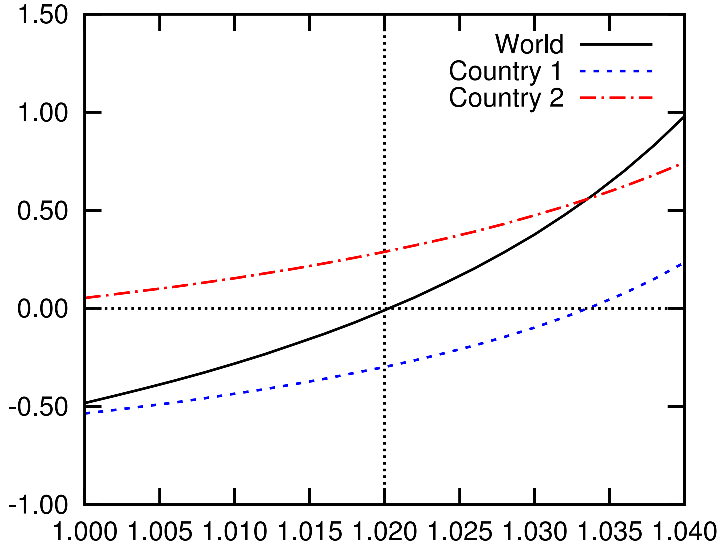

Figure 4: Aggregate savings and equilibrium interest rate

an 'o' sign. This figure shows that global imbalances accumulate quickly. In 2007 the US net foreign asset position was -24% of GDP. To obtain such an imbalance as an equilibrium outcome we need to assume that coefficient of variation of individual income in China is 0.40, 35% higher than that in the U.S.

4 Exchange Rate

In this section we consider an economic environment with two goods. Country produces a differentiated good j and a composite good. The composite good (consumption bundle) in country j is produced using a constant elasticity of substitution (CES) technology:

$$c_t^j = (s_1^j c_{1t}^\rho + s_2^j c_{2t}^\rho)^{1/\rho}. \tag{10}$$

Elasticity of substitution between goods is:

$$e \equiv \frac{1}{1-\rho}.$$

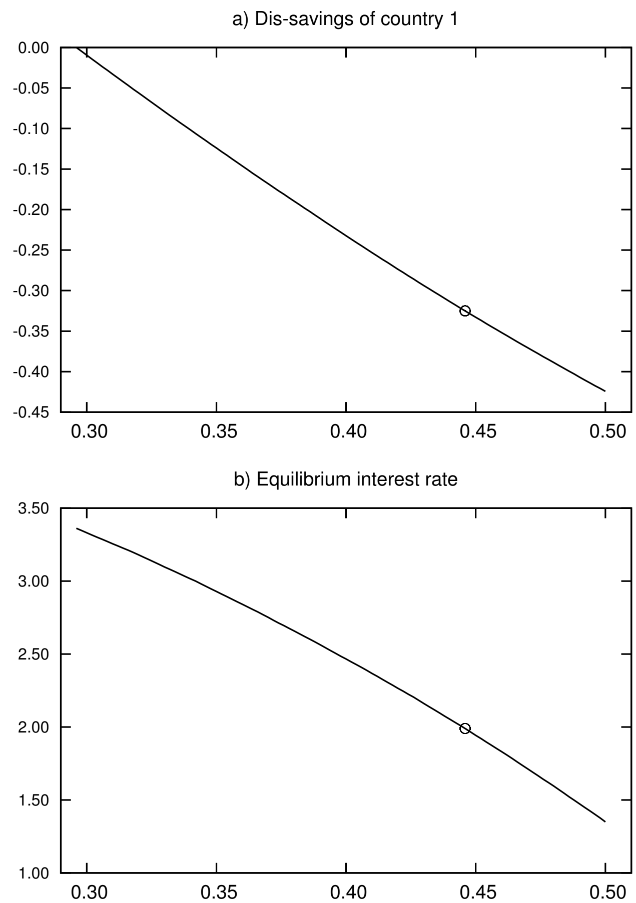

Figure 5: Global imbalances and world interest rate

We impose that preferences of the two countries are symmetric: $s_1^1 = s_2^2 = \omega, s_2^1 = s_1^2 = 1 - \omega$. Further, we assume that consumers exhibit *consumption home bias*: $\omega > 0.5$. This could be a consequence of costly trade as shown in Obstfeld & Rogoff (2000). Modelling costly trade, however, does not add to the insights made here. As will be shown later the exchange rate depends on the equilibrium financial flows but not on how they were generated.

Let p_{jt} denote the international price of good j. Then the price index in country j is:

$$Q_t^j \equiv [(s_1^j)^{\varepsilon} p_{1t}^{1-\varepsilon} + (s_2^j)^{\varepsilon} p_{2t}^{1-\varepsilon}]^{1/(1-\varepsilon)}. \tag{11}$$

The real exchange rate is:

$$Q_t \equiv Q_t^1 / Q_t^2. \tag{12}$$

Budget constraint of a household in country j then is:

$$p_{1t} c_{1t}^j + p_{2t} c_{2t}^j + Q_t^j a_{t+1}^j / R_{t+1}^j = p_{jt} y_t^j + Q_t^j a_t^j.$$

Because the consumption aggregator has constant returns to scale: $Q_t^j c_t^j = p_{1t} c_{1t}^j + p_{2t} c_{2t}^j$ where c_t^j denotes consumption of country j's composite good. Then the budget constraint can be re-written as follows:

$$c_t^j + a_{t+1}^j / R_{t+1}^j = y_t^j p_{jt} / Q_t^j + a_t^j,$$

where c_t^j, a_t^j are measured in units of country j's consumption aggregate. Households in country j are subject to a borrowing limit

$$a_{t+1}^j \geqslant -B \cdot (p_{tj} y_j / Q_t^j). \tag{13}$$

That is a household can borrow up to B average annual incomes.

In the integrated world economy market clearing conditions are: $\forall t$,

$$Q_t^1 A_t^1 + Q_t^2 A_t^2 = 0, \tag{14a}$$

$$c_{jt}^1 + c_{jt}^2 = Y_j. \tag{14b}$$

The homotheticity built into the model can be exploited as follows. Consider a stationary competitive equilibrium and a solution $\{\rho_c^j(a,y), \rho_a^j(a,y)\}$ to the optimization problem of a household in country j for a given price system $\{p_1, p_2, R\}$. Choose a different price system: $\{\tilde{p}_1, \tilde{p}_2, R\}$. If $\tilde{p}_1/\tilde{Q}^1 = \lambda p_1 / Q^1$ then $\{\lambda \rho_c(a,y), \lambda \rho_a(a,y)\}$ is a solution to the household's optimization problem under the modified price system. This fact implies that a CE in the two-good economy can be constructed from a CE in the one-good economy by appropriately choosing p_1/p_2. The algorithm is described in appendix C. This appendix also contains a proof that the algorithm delivers a unique choice of p_1/p_2 for each candidate R.

4.1 Analytical Results

Consider a stationary CE with price system $\{p_1, p_2, R\}$. Let Q^j denote the implied price of consumption aggregate in country j. Let $N^j \equiv A^j(1-1/R)$ denote the net factor income of country j. Then the world economy can be summarized by the following equilibrium conditions:

$$C^1 = p_1 Y_1/Q^1 + N^1, \tag{15a}$$

$$C^2 = p_2 Y_2/Q^2 + N^2, \tag{15b}$$

$$p_1/p_2 = s/(1-s)(c_1^1/c_2^1)^{-1/e}, \tag{15c}$$

$$p_1/p_2 = (1-s)/s(c_1^2/c_2^2)^{-1/e}, \tag{15d}$$

$$0 = Q^1 N^1 + Q^2 N^2, \tag{15e}$$

$$Y_1 = c_1^1 + c_1^2, \tag{15f}$$

$$Y_2 = c_2^1 + c_2^2, \tag{15g}$$

For any $N^1 = -N^2/Q$ we can solve this system for $p_1, p_2, c_1^1, c_2^1, c_1^2, c_2^2$.[6] An analytical solution can be computed in the "symmetric" case with $N^1+N^2 = 0, Y_1 = Y_2 = Y$:

$$c_1^1 = c_2^2 = xY, \quad c_2^1 = c_1^2 = (1-x)Y, \qquad p_1 = p_2 = 1, \tag{16}$$

where

$$x \equiv \frac{\omega^\varepsilon}{\omega^\varepsilon + (1-\omega)^\varepsilon} \geqslant 0.5$$

is the relative share of spending devoted to domestic good. In all other cases we can compute a log-linear approximation to the relation between Q and N^1, N^2 using the "symmetric" solution as an approximation point.

Proposition 2. *In the vicinity of the symmetric CE the relation between Q and N^1 is:*

$$\frac{C}{Y}\left[e((2x-1)^{-2}-1)+1-(2x-1)^{-1}\right] ln(Q) = 2\frac{N^1}{Y} + O(\|N^1/Y\|^2)$$

where C is the aggregate consumption level in the symmetric setting and $O(\|N^1/Y\|^2)$ is a second-order error term.

[6] One equation is redundant by the Walras law. So, it is a 6-dimensional nonlinear equation system.

Proof. See appendix B. □

The left-hand side is the change in the net consumption. The right-hand side is the change in net factor (or financial) income. First, when N^1/Y changes by 1% then N^2/Y must change by the same percentage in the opposite direction. So, the relative wealth changes by $2N^1/Y$. The impact of N on the exchange rate is inversely related to the level of demand measured by C/Y. If demand is high then we need large changes in financial income to affect it and the exchange rate. The term $e[(2x-1)^{-2}-1]$ is the elasticity of the relative demand C^1/C^2 with respect to the real exchange rate Q. It is always positive. The term 1 reflects the impact on the relative cost of country consumption bundles Q. The term $(2x-1)^{-1} > 1$ is the elasticity of the relative income with respect to the real exchange rate Q. When Q increases the relative income of country 1 also increases. The latter must lead to an even larger increase in QC^1/C^2 to match an increase in relative financial income.

Corollary 1. *N^1 and Q are positively related if and only if*

$$e \geqslant \frac{(2x-1)^2}{2x}.$$

The relation between N^1 and Q could be positive or negative. It is likely to be positive if elasticity of substitution between goods e is high or the share of spending devoted to domestic good x is high. Figure 6 shows the approximate (log-linear) and the exact (numerically computed non-linear) relation. The approximation is fairly accurate but it deteriorates quickly as e increases.

We can now interpret this result. Because both goods are traded we can no longer take autarky as the pre-integration stage. So, instead we contrast a developed economy, *e.g.* European union, and China. Baring economic size considerations, the real exchange rate between the U.S. and the E.U (country 2) should be close to 1 as in this case we have $N^1 \approx 0$. If we consider the U.S. vs China (country 2) the real exchange rate should be well below 1 because $N^1 \ll 0$. That is Chinese currency should be valued more relative to other economies. The intuition for this result is that

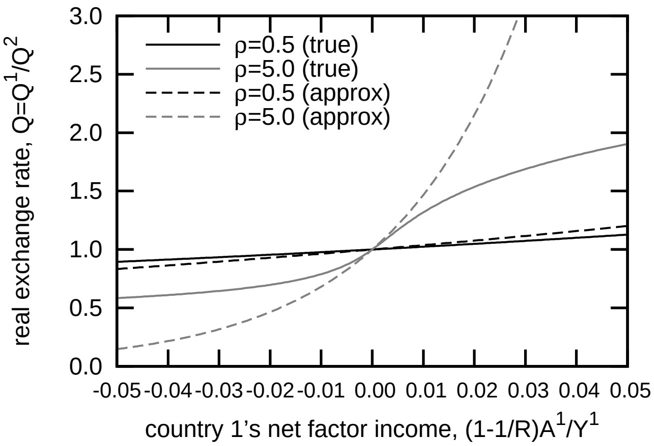

Figure 6: Relation between wealth accumulation and real exchange rate. Dashed lines denote log-linear approximation around the symmetric CE.

wealth accumulation, via higher financial income, drives up demand and price of domestic goods in China. The size of this effect can be estimated using proposition 2. Motivated by the arguments and evidence reported in Obstfeld & Rogoff (2000) we use $e = 5$ and $x = 0.85$ that corresponds to the share of domestic goods in total spending in the U.S. These numerical values give us $ln(Q) \approx 4.78N^1/Y$. N^1/Y for the U.S. in 2012 was -1.8% we get that the real exchange rate of the U.S. should depreciate 8.6% against the world.

5 Conclusions

We used a 2-country version of the model presented in Huggett (1993) to show that global imbalances can easily arise from difference in idiosyncratic risk. Thus, if income volatility in China were only 10% higher than in the U.S. we would observe global imbalances of 7% of the U.S. GDP. The world interest rate would be depressed by 40 basis points. However, we need to assume that income volatility in China is 35% higher to match the evolution of the short term interest rate. In this case global imbalances are 32.5% of the U.S. GDP, larger than 24% observed in the data.

We extend the model to a two-good setting and compute the effect that global imbalances should have on exchange rates. We find that the observed imbalance between the U.S. and China should translate into a 8.6% appreciation of the Chinese real exchange rate.

References

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A Proofs

[TBW]

B Exchange rate derivations

[TBW]

C Computation algorithm for the model with two goods

The algorithm consists of the following steps.

1. Fix some interest rate R and solve household problem for country 1 and country 2 under the price system $\{p_1, p_2, R\} = \{1, 1, R\}$. Let $\{\rho_c^j(a, y), \rho_a^j(a, y)\}$ denote the solution to the household problem in country j.

2. Compute the aggregate savings A^j and consumption C^j for each country.

3. If $A^1 + A^2 = 0$ then set $p^1 = p^2 = 1$ and stop, otherwise continue to the next step.

4. Consider $p_1/p_2 = q > 0$. The optimal policy for country j under the price system $\{q, 1, R\}$ is $\lambda^j \times \{\rho_c^j(a, y), \rho_a^j(a, y)\}$ where:

$$\lambda^1(q) = \left[\frac{s^e + (1-s)^e}{s^e + (1-s)^e q^{e-1}}\right]^{\frac{1}{1-e}},$$

$$\lambda^2(q) = \left[\frac{(1-s)^e + s^e}{(1-s)^e q^{1-e} + s^e}\right]^{\frac{1}{1-e}}.$$

Choose $q > 0$ so that the financial market clears: $Q(q)\lambda^1(q)A^1 + \lambda^2(q)A^2 = 0$. Lemma 1 shows that there exists unique q that achieves this.

5. Check if the market for good 1 clears:[7]

$$\underbrace{\frac{s^e}{[s^\rho + (1-s)^\rho q^{\rho-1}]^{1/\rho}} C^1}_{\text{country 1's demand for good 1}} + \underbrace{\frac{(1-s)^e}{[(1-s)^\rho + s^\rho q^{\rho-1}]^{1/\rho}} C^2}_{\text{country 2's demand for good 1}} - Y_1 \in [-\varepsilon, +\varepsilon].$$

If it clears then stop. Otherwise, return to step 1.

Lemma 1. *For each $R \in [R^1_{aut}, R^2_{aut}]$ there is a unique q for which the financial market clears.*

Proof. At the equilibrium interest rate A^1 and A^2 must be of the opposite sign. This is true if and only if $R \in [R^1_{aut}, R^2_{aut}]$. This insures that one analyzes only viable interest rates.

Then by direct differentiation it can be shown that $d(Q\lambda^1)/dq > 0$ and $d(\lambda^2)/dq < 0$. If $A^1 < 0 < A^2$ ($A^1 > 0 > A^2$) then $Q\lambda^1 A^1 + \lambda^2 A^2$ is a strictly decreasing (increasing) function of q. □

[7] By Walras law if the financial market and the market for good 1 clear then the market for good 2 clears automatically.